On the partial sums and Marcinkiewicz-Fejér means with respect to one- and two-dimensional Walsh-Fourier Series on the one-parameter martingal Hardy spaces

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Unlike the classical theory of Fourier series which deals with decomposition of a function into continuous waves, the Walsh functions are "rectangular waves". Such waves have already been used frequently in the theory of signal transmission, filtering, image enhancement and digital signal processing.

The problems we have studied in this PhD thesis are central to Mathematical Analysis. They involve techniques which have been developed a great deal during the last three decades.

In this PhD thesis we are dealing with the boundedness of partial sums and Marcinkiewicz-Fejér (Fejér) means with respect to one- and two-dimensional Walsh-Fourier series on the one parameter martingale Hardy spaces.

This thesis is focus to achieve the following main results:

· To find estimation of convergence and divergence of the subsequences of partial sums of the one-dimensional Walsh-Fourier series on the martingale Hardy $H_p(G)$ spaces, when $0 < p \leq 1$.

· To find necessary and sufficient conditions for the modulus of continuity of Hardy spaces of $H_p(G)$ martingales, for which subsequences of partial sums of the one-dimensional Walsh-Fourier series convergence on the martingale Hardy $H_p(G)$ spaces, when $0 < p \leq 1$.

· To find estimation of convergence and divergence of the subsequences of Fejér means of the one-dimensional Walsh-Fourier series on the martingale Hardy $H_p(G)$ spaces, when $0 < p \leq 1/2$.

· To find necessary and sufficient conditions for the modulus of continuity of Hardy spaces of $H_p(G)$ martingales, for which subsequences of Fejér means of the one-dimensional Walsh-Fourier series convergence on the martingale Hardy $H_p(G)$ spaces, when $0 < p \leq 1/2$.

· To prove strong convergernce of one-dimensional Fejér means with respect to Walsh system on the martingale Hardy $H_p(G)$ spaces, when $0 < p \leq 1/2$.

· To prove strong convergernce of diagonal partial sums with respect to two-dimensional Walsh-Fourier series on the martingale Hardy $H_p(G^2)$ spaces, when $0 < p < 1$.

· To prove strong convergernce of Marcinkiewicz-Fejér means with respect to two-dimensional Walsh-Fourier series on the martingale Hardy $H_{2/3}(G^2)$ space.

· To find necessary and sufficient conditions for the modulus of continuity of Hardy spaces of $H_{2/3}(G^2)$ martingales, for which subsequences of Marcinkiewicz-Fejér means of the two-dimensional Walsh-Fourier series convergence in $H_{2/3}(G^2)$ space.